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Montana K-12 Mathematics Content Standards and Performance Descriptors

A Vision for Montana Mathematics

The world as we know it is changing at an ever increasing pace. The teaching of mathematics in Montana's public schools needs to be flexible enough to deliver rigorous material that continues to be relevant to the changing lives of our students. In that vein, Montana teachers are challenged to envision the world not as we know it today, but the world our students will be living in tomorrow.

Envision a classroom where instruction is focused on the *big* ideas of mathematics. On a daily basis, students are expected to engage, interact, collaborate, explain and excel. Envision the powerful students such an atmosphere will create—students who are active, excited, curious, and confident; students who *learn*. In this classroom, mathematics is more than just content to be studied; it is an activity to be enjoyed.

There are many aspects of our students' school experience that are outside of our control. However, we do have influence over the mathematics we teach and how we teach it. Montana's mathematics teachers are first class. They are innovators. The standards set forth in this document are of the same quality. To bring them to life requires that Montana educators do what they do best: innovate, challenge and achieve.

Implementing the Vision

The Montana content standards for mathematics are not about mandating curriculum or recommending specific courses in Montana's schools. Instead, they are about preparing students to work and live successfully in a society that is increasingly technical, global and multicultural. The Board of Public Education has set high expectations for the performance of Montana students at all levels; it is the responsibility of local communities and districts to determine the path for their students to achieve the goals set out in this document.

Components of the Standards Document

In order to use this document effectively, it is essential to understand the distinctions between and intended purpose of its various components.

Content Standards: The four mathematics content standards indicate what all students should know, understand, and be able to do in mathematics. Their purpose is to guide the mathematics curriculum and to communicate the breadth of the mathematics to be taught to all students. A district's curriculum should be designed so that learning encompasses all four standards.

Benchmarks: The benchmarks define expectations for students' mathematical knowledge and skills along a developmental continuum. They define expectations for proficient students at the end of grade 4, end of grade 8, and upon graduation. Their purpose is to state clearly and specifically what the students should know and be able to do within each content standard. A district's curriculum should include the entire progression of knowledge contained in the benchmarks.

Performance Descriptors: Performance descriptors define how well students apply the knowledge and skills they have acquired. They gauge the level to which benchmarks have been attained in terms of range, frequency, facility, depth, creativity and quality. Achievement of curricular goals is assessed by the performance descriptors.

Pursuant to Article X Sect 1(2) of the Constitution of the state of Montana and statutes §20-1-501 and §20-9-309 2(c) MCA, the implementation of these standards must incorporate the distinct and unique cultural heritage of Montana American Indians.

Number Sense and Operation Mathematics Content Standard 1

A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts.

Rationale

Number sense and computational fluency are the foundation for school mathematics and life in a multicultural and quantitative society. Students who have a sense of quantity are fluent with basic facts, perform mental computations, understand that knowing the properties of operations help them solve problems, determine the reasonableness of solutions, and use number to describe their world. The foundation of number sense and operations supports the other content standards.

Benchmarks

End of Grade 4	End of Grade 8	Upon Graduation
1.1 Whole Number Relationships: Demonstrate relationships among whole numbers; identify place value up to 100,000 and compare numbers (e.g., greater than, less than, and equal to).	**1.1 Rational Number Relationships**: Recognize, model, and compare different forms of integers and rational numbers including percents, fractions, decimals, and numbers using exponents and scientific notation.	**1.1 Quantification**: Use multiple notations to perform and interpret the effects of operations on very large and very small numbers with and without technology.
1.2 Estimation and Operations: Estimate sums, differences, products, and quotients when solving problems. Add, subtract, multiply (three-digit by two-digit factors), and divide (two-digit dividends by one-digit divisors) to solve problems. Demonstrate fluency with basic facts.	**1.2 Estimation and Reasonableness**: Select and apply appropriate estimation strategies to judge the reasonableness of solutions to problems including those computed on a calculator. Demonstrate correct use of order of operations.	**1.2 Estimation and Accuracy**: Identify situations where estimation is appropriate and determine the degree of accuracy needed for a given problem situation (and the appropriate precision in which to report answers).
1.3 Whole Number Concepts: Develop multiplication and division concepts, apply number and operation models and strategies, and reason and justify using properties of operations.	**1.3 Number Theory**: Use number theory concepts such as prime factorization, greatest common factor, and least common multiple in problem situations.	**1.3 Equivalence with Multiple Notation**: Given a representation of a number or expression, find equivalent representations using multiple notations (e.g., $x^{1/2}$ vs. $\sqrt{x}$ and visual representation of multiplying binomials).

End of Grade 4	End of Grade 8	Upon Graduation
1.4 Common Fractions and Decimals: Identify and model common fractions such as, tenths, fourths, thirds, and halves; and decimals such as money and place value to 0.001; and recognize and compare equivalent representations.	**1.4 Rational Number Operations**: Compute fluently and solve multi-step problems using integers, fractions, decimals, and numbers in exponential form.	**1.4 Properties of Numbers and Number Systems**: Analyze and apply the properties of numbers and number systems.
1.5 Length, Time, and Temperature: Select and apply appropriate standard units and tools to measure length, time, and temperature within relevant scientific and cultural situations.	**1.5 Metric and Standard Measurement**: Use metric and standard units of measurement in relevant scientific and cultural situations, compare and convert within systems, and use appropriate technology.	**1.5 Modeling Relationships and Change**: Identify givens and unknowns in familiar and unfamiliar situations (e.g., finance, culture, and nature) and describe relationships between variables.
	1.6 Proportional Reasoning: Understand and apply proportional relationships to model real world situations and to solve problems involving rates, ratios, proportions, percents, and direct variation.	

Data Analysis Mathematics Content Standard 2

A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts.

Rationale

Data analysis and statistical literacy pertain to all aspects of daily life within multiple cultures. As consumers of information, students who analyze data to make decisions and predictions are better prepared to be responsible citizens. Students who understand and apply basic concepts of probability and make connections to data analysis build strong quantitative reasoning for productive personal and professional lives.

Benchmarks

End of Grade 4	End of Grade 8	Upon Graduation
2.1 Representing Data: Collect, represent, and organize data in tables, dot plots, bar graphs, pictographs, and stem and leaf plots using technology when appropriate.	**2.1 Representing and Comparing Data:** Collect data from a variety of contexts (e.g., science, history, and culture). Organize and represent data in box plots, scatter plots, histograms, and circle graphs using technology when appropriate.	**2.1 Representing and Analyzing Data**: Select, create, and compare graphical or numerical representations of data sets using technology when appropriate. Reason about distributions using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation).
2.2 Evaluating Data: Solve problems and make decisions using data descriptors such as minimum, maximum, median, and mode within scientific and cultural contexts when relevant.	**2.2 Evaluating Data and Making Conjectures:** Interpret, analyze, and evaluate data using mean, median, range, and quartiles to identify trends and make decisions and predictions about data within scientific and cultural contexts when relevant.	**2.2 Evaluating Validity**: Evaluate the validity of reports based on collected and/or published data by considering the source of the data, the design of the study, and the way data are displayed, analyzed, and interpreted.

End of Grade 4	End of Grade 8	Upon Graduation
2.3 Likelihood of Events: Describe events from multicultural contexts as likely or unlikely and discuss the degree of likelihood using words such as certain, equally likely, and impossible.	**2.3 Finding Probability and Predicting**: Create sample spaces and simulations from events found in different cultures, determine experimental and theoretical probabilities, and use probability to make predictions.	**2.3 Rules of Probability and Expected Value**: Make, evaluate, and justify decisions based on probabilities in multicultural problem situations (e.g., finding expected value and using rules of probability).
		2.4 Counting Methods: Use technology as needed to determine the possible number of outcomes for an event or compound event using the fundamental counting principle, permutations, combinations, and other systematic counting methods.
		2.5 Curve Fitting: Model two-variable data using curve fitting with and without technology. Write an equation for a given model and decide when or if predictions based on this equation are valid.

Geometric Reasoning Mathematics Content Standard 3

A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts.

Rationale

Geometric reasoning complements the study of number, operations, and probability models. Students who have a sense of space analyze two- and three-dimensional shapes and their properties and relationships, and can make connections within mathematics. Geometric reasoning helps students appreciate and value mathematics and make connections to their world through multiple cultural contexts.

Benchmarks

End of Grade 4	End of Grade 8	Upon Graduation
3.1 Two-Dimensional Attributes: Describe, compare, and analyze attributes of two-dimensional shapes.	**3.1 Properties of Solids and Figures**: Define, classify and compare properties of solids and plane figures, including lines and angles.	**3.1 Conjectures and Inductive Reasoning:**, Formulate and evaluate conjectures about geometric objects and their properties, with and without technology, applying inductive reasoning when appropriate.
3.2 Three-Dimensional Attributes: Describe attributes of three-dimensional shapes such as cubes and other rectangular prisms, pyramids, cylinders, cones, and spheres.	**3.2 Congruence and Similarity**: Use spatial reasoning to determine congruence, similarity, and symmetry of objects in mathematics, art, science, and culture.	**3.2 Applications of Geometric Models**: Use spatial reasoning and geometric models to solve problems with and without technology in the contexts of art, science, and culture.
3.3 Basic Transformations: Use spatial reasoning to identify slides and flips of congruent figures within cultural and artistic contexts.	**3.3 Transformations including Dilations:** Define, identify, and execute transformations including translations, rotations, reflections, and dilations with appropriate technology.	**3.3 Multiple Geometric Approaches**: Identify, analyze, and use transformational, coordinate, and synthetic geometric approaches to solve problems.

End of Grade 4	End of Grade 8	Upon Graduation
3.4 Linear Measurement: Estimate and measure linear attributes of objects in metric units such as centimeters and meters and customary units such as inch, foot, and yard.	**3.4 Angles, Surface Area, and Volume**: Measure and compute angles, perimeter, area, surface area, and volume including the use of formulas and choosing appropriate units.	**3.4 Indirect Measurement:** Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships.
3.5 Area and Perimeter: Define and determine area and perimeter of common polygons using concrete tools such as grid paper, objects, or technology and justify the strategy used.	**3.5 Justifying Relationships**: Develop informal arguments to verify geometric relationships and solve problems such as an informal justification of the Pythagorean Theorem in a variety of contexts.	**3.5 Methods of Proof:** Establish the validity of geometric conjectures using deductive reasoning, indirect proof, and counterexamples, and critique arguments made by others.

Algebraic and Functional Reasoning Mathematics Content Standard 4

A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts.

Rationale

The study of algebra and functions opens doors and expands opportunities in numerous 21st century careers throughout many cultures. Students who generalize patterns and represent relationships in multiple ways develop significant understandings of mathematics and the use of quantitative reasoning in other disciplines. Algebra and functions are powerful tools for modeling real world relationships and making informed decisions.

Benchmarks

End of Grade 4	End of Grade 8	Upon Graduation
4.1 Patterns and Relations: Describe, extend, and make generalizations about geometric or numeric patterns.	**4.1 Representing and Generalizing Patterns:** Create and use tables, graphs or diagrams, symbolic expressions, and verbal descriptions to represent, analyze, and generalize a variety of patterns involving numbers and operations.	**4.1 Representing Functions:** Represent functions in a variety of ways including tables, graphs or diagrams, verbal descriptions, and symbolic expressions in recursive and explicit form. Justify the choice of an appropriate form for solving a given problem.
4.2 Symbols and Expressions: Use letters, boxes, or symbols to represent numbers in simple expressions or equations to demonstrate a basic understanding of variables.	**4.2 Linear Functions**: Identify linear and non-linear functional relationships and contrast their properties using tables, graphs, or equations with appropriate technology.	**4.2 Variables and Parameters:** Determine the appropriate symbolic representation of a given contextual situation (e.g., variables and parameters in equations, inequalities, functions, and matrices).

End of Grade 4	End of Grade 8	Upon Graduation
4.3 Properties of Number and Operation: Use number patterns to investigate properties of numbers such as even or odd and properties of operations such as commutative, associative, distributive, and the multiplicative and additive identities.	**4.3 Multi-step equations and inequalities:** Use number properties and inverse operations to solve multi-step equations and inequalities involving a single variable.	**4.3 Solving Systems of Equations and Inequalities**: Solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context.
4.4 Equivalent Expressions: Develop an understanding of equivalence by expressing numbers, measures, and numerical expressions involving operations in a variety of ways.	**4.4 Equivalent Algebraic Expressions**: Recognize, simplify, and generate equivalent forms of algebraic expressions, justifying each step with properties of operations.	**4.4 Families of Functions and Transformations**: Analyze the effects of transformations on families of functions and recognize their characteristics. Represent and use functions in equivalent forms to identify and perform transformations.
4.5 Numerical Modeling with Manipulatives: Model problem situations with manipulatives or technology and use multiple representations such as words, pictures, tables, or graphs to draw conclusions using cultural contexts when relevant.	**4.5 Linear Modeling**: Identify and compute rate of change/slope and intercepts from equations, graphs, and tables; model and solve contextual problems involving linear proportions or direct variation using cultural contexts when relevant.	**4.5 Analyzing and Conjecturing with Models:** Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using cultural contexts when relevant.

Montana K-12 Mathematics Performance Descriptors – A Profile of Four Levels

The Mathematics Performance Descriptors define how well students' perform at four performance levels: advanced, proficient, nearing proficiency, and novice. These profiles describe students as they apply the knowledge and skills defined in the benchmarks and found in the "Benchmarks At-A-Glance" document for End of Grade 4, End of Grade 8, and Upon Graduation.

Advanced	Proficient	Nearing Proficiency	Novice
A student at the advanced level in Mathematics demonstrates superior performance. He/she:	A student at the proficient level in Mathematics demonstrates solid academic performance. He/she:	A student at the nearing proficiency level in Mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficiency. He/she:	A student at the novice level in Mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental for proficiency. He/she
• gives responses that exhibit advanced understanding of the problem or situation presented, • consistently demonstrates advanced conceptualization • makes connections within and between the mathematical and real world, • applies more than one process and uses multiple representations to determine solutions accurately, and • clearly communicates and justifies reasoning and structure of solutions.	• gives responses that exhibit clear understanding of the problem or situation presented, • makes connections within the mathematical and/or real world, • determines a process and sufficiently communicates correct problem solving strategies through relevant representations, • has occasional errors but these do not interfere with appropriate strategies, and • has reasonable and well-supported solutions.	• gives responses that exhibit some understanding of the problem or situation presented, • makes rudimentary connections between the mathematical and/or real world, • struggles to communicate effectively, • uses limited evidence of representations to show understanding, • has some basic reasoning skills that are apparent but uses insufficient computational skills and problem solving strategies, and • has frequent errors and lack of structure that detract from mathematical knowledge and skills.	• gives responses that exhibit significant difficulty in understanding basic concepts, reasoning, implementing problem solving strategies, and making connections, • severely lacks basic skills, representation, structure, and process development, • attempts to solve problems, • has substantial errors, and • lacks communication skills that hinder student's progress.

These profiles apply to the "Benchmarks At-A-Glance" document for End of Grade 4, End of Grade 8, and Upon Graduation.

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Montana K-12 Mathematics Performance Descriptors – Benchmarks At-A-Glance

End of Grade 4	End of Grade 8	Upon Graduation
Number Sense and Operations		
1.1 whole number relationships; 1.2 estimation and operations; 1.3 whole number concepts; 1.4 common fractions and decimals; 1.5 length, time, and temperature.	1.1 rational number relationships; 1.2 estimation and reasonableness; 1.3 number theory; 1.4 rational number operations; 1.5 metric and standard measurement; 1.6 proportional reasoning.	1.1 quantification; 1.2 estimation and accuracy; 1.3 equivalence with multiple notation; 1.4 properties of numbers and number systems; 1.5 modeling relationships and change.
Data Analysis		
2.1 representing data; 2.2 evaluating data; 2.3 likelihood of events.	2.1 representing and comparing data; 2.2 evaluating data and making conjectures; 2.3 finding probability and predicting.	2.1 representing and analyzing data sets; 2.2 evaluating validity; 2.3 rules of probability and expected value; 2.4 counting methods; 2.5 curve fitting.
Geometric Reasoning		
3.1 two-dimensional attributes; 3.2 three-dimensional attributes; 3.3 basic transformations; 3.4 linear measurement; 3.5 area and perimeter.	3.1 properties of solids and figures; 3.2 congruence and similarity; 3.3 transformations including dilations; 3.4 angles, surface area, and volume; 3.5 justifying relationships.	3.1 conjectures and inductive reasoning; 3.2 applications of geometric models; 3.3 multiple geometric approaches; 3.4 indirect measurement; 3.5 methods of proof.
Algebraic and Functional Reasoning		
4.1 patterns and relations; 4.2 symbols and expressions; 4.3 properties of number and operation; 4.4 equivalent expressions; 4.5 numerical modeling with manipulatives.	4.1 representing and generalizing patterns; 4.2 linear functions; 4.3 multi-step equations and inequalities; 4.4 equivalent algebraic expressions; 4.5 linear modeling.	4.1 representing functions; 4.2 variables and parameters; 4.3 solving systems of equations and inequalities; 4.4 families of functions and transformations; 4.5 analyzing and conjecturing with models.